Department of Computer Science
University of Pretoria

COS284

Tutorial 4

Due: 3rd September 2015

Surname: ____________________________________________

First name: ____________________________________________

Student number: ____________________________________________

Marked by: ____________________________________________
Surname: ____________________________________________
First name: ____________________________________________
Student number: ____________________________________________

Instructions

1. Complete this tutorial on paper and bring your completed worksheet to the tutorial session on the due date specified above. Tutorials completed in pencil will not be marked.

2. Note: loose pages will not be accepted! All pages of your tutorial must be stapled together before you come to the tutorial.

3. You must stay for the duration of the tutorial session and peer mark another student’s worksheet, which will be given to you.

4. If you arrive late, you will not be allowed to join the tutorial marking session and you will get 0 for the tutorial.

5. If you do not mark another worksheet, you will not obtain any marks for the tutorial.

6. Maximum mark is 37.

<table>
<thead>
<tr>
<th>Question</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marks:</td>
<td>4</td>
<td>8</td>
<td>25</td>
<td>37</td>
</tr>
</tbody>
</table>

Score:
This tutorial will test your understanding of simple combinational circuits, including the design of such circuits using Karnaugh maps.

1. Consider the following logical circuit.

![Logical Circuit Diagram]

Provide logical expressions for the outputs $N_1$ and $N_0$.

**Solution:** Two marks for each equation; 2 if correct, 1 if slightly flawed

$N_1 = Y_2 + Y_3$

$N_0 = Y_3 \oplus Y_1$

2. Consider the following Karnaugh maps. Indicate the relevant groups on the provided Karnaugh maps. Then write the boolean expression that one can “read” from the selected groups (as a sum of products).

(a)

<table>
<thead>
<tr>
<th>$ab$</th>
<th>00</th>
<th>01</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td></td>
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<td>1</td>
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</table>

**Solution:** One mark for each of the two groups on the diagram; one mark for each of the two terms in the expression.
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>00</td>
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<td>11</td>
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<tr>
<td>10</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Answer: $a'b + b'c$
\textbf{Solution:} One mark for each of the two groups on the diagram; one mark for each of the two terms in the expression.

\begin{center}
\begin{tabular}{c|cccc}
\hline
\textit{cd} & 00 & 01 & 11 & 10 \\
\hline
\textit{ab} & \\
\hline
00 & 0 & 1 & x & 0 \\
01 & 1 & 1 & x & 1 \\
11 & 0 & 0 & 0 & 0 \\
10 & x & x & x & x \\
\hline
\end{tabular}
\end{center}

Answer: $a'b + a'd$

\textbf{OR}

\begin{center}
\begin{tabular}{c|cccc}
\hline
\textit{cd} & 00 & 01 & 11 & 10 \\
\hline
\textit{ab} & \\
\hline
00 & 0 & 1 & x & 0 \\
01 & 1 & 1 & x & 1 \\
11 & 0 & 0 & 0 & 0 \\
10 & x & x & x & x \\
\hline
\end{tabular}
\end{center}

Answer: $a'b + bd$
3. (a) Let $x$ and $y$ each be two-bit unsigned integers ($x_{1}x_{0}$ and $y_{1}y_{0}$, respectively).

Let

$$f(x, y) = \begin{cases} 
1 & \text{if the difference between } x \text{ and } y \text{ is even} \\
0 & \text{otherwise}
\end{cases}$$

Use a Karnaugh map to derive an equation for $f(x, y)$.

**Solution:** One mark for each row of the Karnaugh map ($\frac{1}{2}$ mark if final two columns are swapped); one mark for each of the two terms in the answer.

$$f(x, y) = x'_{0}y'_{0} + x_{0}y_{0}$$
(b) Let $x$ and $y$ each be two-bit unsigned integers ($x_1x_0$ and $y_1y_0$, respectively). Let

$$f(x, y) = \begin{cases} 1 & \text{if the difference between } x \text{ and } y \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

Use a Karnaugh map to derive an equation for $f(x, y)$.

\[
\begin{array}{cccc}
00 & 01 & 11 & 10 \\
00 & & & \\
01 & & & \\
11 & & & \\
10 & & & \\
\end{array}
\]

**Solution:** Two marks for the values in the Karnaugh map (1 mark if columns or rows are swapped, but content is correct according to labels); one mark for each of the two terms in the expression.

\[
\begin{array}{cccc}
00 & 01 & 11 & 10 \\
00 & 0 & 1 & 1 & 0 \\
01 & 1 & 0 & 0 & 1 \\
11 & 1 & 0 & 0 & 1 \\
10 & 0 & 1 & 1 & 0 \\
\end{array}
\]

Answer: $f(x, y) = x'_0y_0 + x_0y_0$

(c) Draw the circuit for $f(x, y)$ from Question b.

**Solution:** 1 mark for every gate (2 AND, 1 OR, and 2 NOTs (which may be indicated as circles on the inputs of the and gates).
Let

\[ f(x, y) = \begin{cases} 
1 & \text{if } x + y < 4 \\
0 & \text{otherwise}
\end{cases} \]

where \( x \) is the unsigned two bit string \( x_1x_0 \) and \( y \) is the unsigned two bit string \( y_1y_0 \).

Derive an equation for \( f(x, y) \) using a Karnaugh map.

**Solution:**

1 mark for correct map labels (1)
1 mark for correct values in each of the 4 rows (4)
1 mark for each correct term in the expression (5)

\[
\begin{array}{c|cccc}
\hline
y_1y_0 & 00 & 01 & 11 & 10 \\
\hline
x_1x_0 & & & & \\
00 & \times & 1 & 1 & \\
01 & 1 & 1 & 0 & 1 \\
11 & 1 & 0 & 0 & 0 \\
10 & \times & 1 & 0 & 0 \\
\hline
\end{array}
\]

Answer: \( x_1'y_1 + x_1'x_0' + y_1'y_0' + x_1'y_0 + x_0'y_1' \)