Department of Computer Science
University of Pretoria

COS284

Tutorial 1

Due: 30th July 2015

Student surname, first name ________________________________

Student number: __________________________________________

Marker surname, first name __________________________________

Marker student number: _____________________________________

Instructions

1. Complete this tutorial on paper and bring your completed worksheet to the tutorial session on the due date specified above. Tutorials completed in pencil will not be marked.

2. Note: loose pages will not be accepted! All pages of your tutorial must be stapled together before you come to the tutorial.

3. You must stay for the duration of the tutorial session and peer mark another student’s worksheet, which will be given to you.

4. If you arrive late, you will not be allowed to join the tutorial marking session and you will get 0 for the tutorial.

5. If you do not mark another worksheet, you will not obtain any marks for the tutorial.

6. Maximum mark is 35.

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Score: ___________________
This tutorial will test your understanding of integer data representation.

1. Convert $1010_{10}$ to binary using the division-remainder method. Show all your working. (4)

Solution:

\[
\begin{array}{c|c}
2 & 1010 \\
2 & 505 \\
2 & 252 \\
2 & 126 \\
2 & 63 \\
2 & 31 \\
2 & 15 \\
2 & 7 \\
2 & 3 \\
2 & 1 \\
\hline
0
\end{array}
\]

Final answer: $1010_{10} = 1111110010_2$

Mark allocation: 3 marks for working (subtract 0.5 for each error), 1 mark for final answer.

2. Convert $1010_{10}$ to octal using the division-remainder method. Show all your working. (3)

Solution:

\[
\begin{array}{c|c}
8 & 1010 \\
8 & 126 \\
8 & 15 \\
8 & 1 \\
\hline
0
\end{array}
\]

Final answer: $1010_{10} = 1762_8$

Mark allocation: 2 marks for working (subtract 0.5 for each error), 1 mark for final answer.
3. (a) What is the range of decimal unsigned integers that can be represented using 12 bits?

**Solution:** 0 to $2^{12} - 1$, that is 0 to 4095

(b) What is the range of decimal unsigned integers that can be represented using 12 trits (0, 1, or 2)?

**Solution:** 0 to $3^{12} - 1$, that is 0 to 531440

4. If you are told that the number $101_r$ to some base $r$ is equivalent to $257_{10}$, what is $r$? Motivate your answer.

**Solution:** $r = 16$, since $16^2 + 16^0 = 257$.

5. (a) Convert 25.82\(_{10}\) to binary using 4 bits to the right of the binary point. Show all your working.

**Solution:**

\[
\begin{align*}
2 \cdot 25 & \quad \text{remainder 1} \\
2 \cdot 12 & \quad \text{remainder 0} \\
2 \cdot 6 & \quad \text{remainder 0} \\
2 \cdot 3 & \quad \text{remainder 1} \\
2 \cdot 1 & \quad \text{remainder 1} \\
0 \times 2 = 1.64: & \quad 1 \times 2^{-1} \\
.64 \times 2 = 1.28: & \quad 1 \times 2^{-2} \\
.28 \times 2 = 0.56: & \quad 0 \times 2^{-3} \\
.56 \times 2 = 1.12: & \quad 1 \times 2^{-4}
\end{align*}
\]

Final answer: 25.82\(_{10}\) \(\approx\) 11001.1101\(_2\)

Mark allocation: 5 marks for working (subtract 0.5 marks for each error), 1 mark for final answer.

Note: It is also acceptable if students use the repeated subtraction method to obtain the answer.

(b) In your binary answer above, some precision is lost due to the restriction of 4 bits to the right of the binary point. By how much does the value in binary differ from the value in decimal? Express the difference in decimal.

**Solution:**

\[
11001.1101_2 = 25 + \frac{1}{2} + \frac{1}{4} + \frac{1}{16} = 25.8125
\]

Therefore the answer in binary is $0.0075_{10}$ smaller than the original value in decimal.
6. Consider a representation of unsigned integers to the base 32 using digits 0 – 9 with A – V to represent the numbers 10 to 31. Convert the number 1001101000001011₂ to a base 32 number by grouping bits. Show your working.

**Solution:**

\[
\begin{array}{cccccccc}
00001 & 00110 & 10000 & 01011 \\
1 & 6 & G & B
\end{array}
\]

Final answer: 1001101000001011₂ = 16GB₃₂

Mark allocation: 3 marks for working, 1 mark for final answer.

7. Add 00001₁₁₀₁₂ to 100₁₁₁₀₀₂ using signed magnitude arithmetic. Show your working.

**Solution:**

The sum is equivalent to subtracting 000₁₁₁₀₀₂ from 000₀₁₁₀₁₂. Since 000₁₁₁₀₀₂ is bigger than 000₀₁₁₀₁₂, we subtract 000₀₁₁₀₁₂ from 000₁₁₁₀₀₂ and negate the answer:

\[
\begin{array}{cccccccc}
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 1
\end{array}\Rightarrow \text{borrows}
\]

\[
\begin{array}{cccccccc}
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1
\end{array}
\]

Final answer: 100₀₁₁₁₀₂

Mark allocation: 1 mark for having the largest magnitude above, 2 marks for magnitude of final answer and 1 mark for correct sign of answer.

8. Show how the sum of 2₅₁₀ and −2₅₁₀ will be calculated using 8-bit two’s complement arithmetic.

**Solution:**

\[
\begin{array}{cccccccc}
2₅₁₀ & 00011001 \\
−2₅₁₀ & 11100111
\end{array}
\]

00000000
9. Subtract the following signed binary numbers as shown using two’s complement arithmetic. Also give the final answer in decimal.

\[ \begin{align*}
00010001 & \quad - 00100000 \\
+11100000 &
\end{align*} \]

Solution:

\[ \begin{align*}
00010001 \\
+11100000 \\
11110001 &= -15_{10}
\end{align*} \]

10. What is the decimal range of signed integers that can be represented using 6-bit two’s complement representation? Also give the range in two’s complement binary representation.

Solution: The range is from \(-32_{10}\) to \(31_{10}\) or from \(100000_{2}\) to \(011111_{2}\)